

Discovery of the Color Degree of Freedom in Particle Physics: a Personal Perspective

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Abstract

I review the main features of the color charge degree of freedom in particle physics, sketch the paradox in the early quark model that led to color, give a personal perspective on the discovery of color and describe the introduction of the gauge theory of color.

1 Introduction

Our present conception of the nature of elementary particles includes fractionally charged quarks that carry a hidden 3-valued charge degree of freedom, “color,” as fundamental constituents of strongly interacting particles (hadrons). The main features of color are (1) it is a hidden 3-valued charge degree of freedom carried by quarks, (2) it can be incorporated into an $SU(3)_{color}$ gauge theory, and (3) the hidden color gauge group commutes with electromagnetism. This third feature requires that the electric charges of quarks are independent of color, which in turn requires the quarks to have fractional electric charges.

Quarks with fractional electric charges were introduced by Murray Gell-Mann [1] and, independently, by George Zweig [2] in 1964. Also in 1964 I introduced color, using parafermi statistics of order 3 [3]. This 3 is the same 3 as the 3 of $SU(3)_{color}$.

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My work was stimulated by the $SU(6)$ theory of Feza Gürsey and Luigi A. Radicati [4] in the same year. Gürsey and Radicati placed the baryons in the symmetric 3-particle representation of $SU(6)$. This produced a paradox: The spin 1/2 quarks must be fermions, according to the spin-statistics theorem, and can only occur in antisymmetric representations. I resolved this paradox in 1964 [3] by suggesting that quarks obey parafermi statistics [5] of order 3, which allows up to 3 particles to be in a symmetric state. As mentioned above, the 3 of the parafermi statistics is the same 3 as the 3 of color $SU(3)$.

Because particles with fractional electric charge had not been observed, several of the early authors chose models with integer quark charges. Such models are unacceptable both theoretically and experimentally. In models with integer quark charges electromagnetism does not commute with color so that color symmetry is broken. Such a model violates the exact conservation of color which is a crucial part of QCD. Integer charges also conflict with experimental evidence coming from the ratio $\sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ as well as from the analysis of jets in high energy hadron collisions.

I emphasize that there are two independent discoveries connected with the strong interactions: (1) color as a charge—analogue to electric charge in electromagnetism, and (2) color as a gauge symmetry—analogue to the $U(1)$ symmetry of electromagnetism.

The gauge symmetry of a theory is intimately connected with the quantities that are observable in the theory. In the context of parastatistics if only currents such as

$$[\bar{\psi}(x), \gamma^\mu \psi(x)] \tag{1}$$

are observable, then the gauge symmetry is $SU(3)$. With additional observables such as

$$[\psi(x), \gamma^\mu \psi(x)] \tag{2}$$

the symmetry is $SO(3)$.

For the parafermi theory of quarks we choose only baryon number zero currents, so that only the currents of Eq.(1) are allowed. Currents such as Eq.(2) have non-zero baryon number and are not allowed. Thus the parafermi theory must be associated with the symmetry $SU(3)$. We can make this explicit, following Oscar

Klein [6], by transforming the Green components of the parafields to sets of normal fields. The choice of currents with zero baryon number leads to explicit $SU(3)$ symmetry for the normal quark fields. To summarize, the choices of observables and of gauge symmetry are directly related.

The parastatistics of H.S. Green cannot be gauged because the commutation rules for the Green components with equal values of the Green index are not the same as the commutation rules for Green components with unequal values of the Green index. Kenneth Macrae and I [7] showed how to modify Green's parastatistics so that it can be gauged by reformulating parastatistics with Grassmann numbers. Further, we showed that using Grassmann numbers that obey an $SU(N)$ ($SO(N)$) algebra leads to an $SU(N)$ ($SO(N)$) gauge theory.

In summary, the full understanding of color emerged from the work of Greenberg in 1964, and the work of Nambu in 1965 and of Han and Nambu in 1965. As is often the case in the development of new theories, neither got everything in final form at the beginning.

2 Influences leading to the discovery of the hidden 3-valued color charge degree of freedom

Here I describe the disparate influences that led me to introduce the color charge degree of freedom. In the 1950's and early 1960's I was struck by the success of very simple ideas in bringing order to the newly discovered "strange" particles. In the same period, under the influence of my thesis advisor, Arthur Wightman, I was learning sophisticated mathematical techniques based on quantum field theory. My PhD thesis on the asymptotic condition in quantum field theory gave a formalization of the Lehmann, Symanzik, Zimmermann (LSZ) scattering theory. I used the theory of operator-valued distributions and gave proofs of properties of LSZ scattering theory that were mathematically rigorous according to the standards of that period.

I became interested in the theory of identical particles as a graduate student in the 1950's. I wondered why only bosons and fermions occur in Nature, as well as what other possibilities might exist. Although I did not see his paper at the time,

H.S. Green had introduced a generalization of each type of statistics in 1953. He generalized bose (fermi) statistics to parabose (parafermi) statistics of integer order p . Green replaced the usual bilinear commutation or anticommutation relations for bose and fermi statistics by trilinear relations. He found solutions of his trilinear relations using an ansatz. Expand a field q in p “Green” components that (for the parafermi case) anticommute when the Green indices are the same but commute when the Green indices are different,

$$[q^{(\alpha)}(x), q^{(\alpha)\dagger}(y)]_+ = \delta(\mathbf{x} - \mathbf{y}), x^0 = y^0, \quad (3)$$

$$[q^{(\alpha)}(x), q^{(\beta)\dagger}(y)]_- = 0, x^0 \neq y^0. \quad (4)$$

(For the parabose case interchange commutator and anticommutator.) For parafermi (parabose) statistics of order p at most p identical particles can be in a symmetric (antisymmetric) state.

As mentioned above, Klein gave a recipe for converting fields that anticommute (commute) into fields that commute (anticommute). Schematically, his transformation is

$$Q^{(\alpha)}(x) = K_{(\alpha)} q^{(\alpha)}(x), \quad (5)$$

$$K_{(\alpha)} |0\rangle = |0\rangle. \quad (6)$$

The Klein transformation converts the anomalous anticommutation (commutation) relations to the normal ones.

Albert M.L. Messiah and I worked together on generalizations of the usual bose and fermi quantum statistics in 1962-1964. We showed that any representation of the symmetric group for identical particles is compatible with quantum mechanics in the context of first-quantized quantum theory [8]. We also worked out the branching rules for changes in the number of identical particles. We formulated parastatistics without using Green’s ansatz (for the case with the usual Fock vacuum) in the context of second-quantized quantum field theory [9]. In addition, we derived the selection rules for interactions that change the number of identical particles. This work prepared me to address the paradox in the quark model of baryons that arose in 1964.

The year 1964 was the crucial year for the discovery of both quarks and color. Quarks were suggested independently by Gell-Mann and by Zweig. Gell-Mann’s

quarks resembled what we now call current quarks. Zweig's quarks, which he called aces, deuces and treys, resembled what we now call constituent quarks. In early 1964 when I first heard about rumors of the idea of quarks I wondered why only the combinations qqq and $\bar{q}q$ occurred in nature. In the original models there was no reason for this.

The paradox concerning the quarks in baryons arose in the $SU(6)$ theory of Gürsey and Radicati. They generalized an idea of Wigner from 1937. Wigner had combined the $SU(2)_I$ of isospin with the $SU(2)_S$ of spin to make an $SU(4)$ and used this $SU(4)$ to classify nuclear states and derive relations for their energy levels. With a larger symmetry group he found more relations among the energy levels. Gürsey and Radicati combined the $SU(3)_f$ of the three quark flavors in the original quark model with $SU(2)_S$ to get an $SU(6)$ that they used to classify particle states.

The $SU(6)$ theory considers a quark as a

$$\mathbf{3} \sim (u, d, s) \text{ in } SU(3)_f \quad (7)$$

and the spin

$$\mathbf{1/2} \sim (\uparrow, \downarrow) \text{ in } SU(2)_S. \quad (8)$$

Gürsey and Radicati combined these as a

$$\mathbf{6} \sim (\mathbf{3}, \mathbf{1/2}) \sim (u_\uparrow, u_\downarrow, d_\uparrow, d_\downarrow, s_\uparrow, s_\downarrow) \text{ in } SU(6). \quad (9)$$

We can also decompose the quark under

$$SU(6) \rightarrow SU(3)_f \times SU(2)_S, \quad (10)$$

$$\mathbf{6} \rightarrow (u, d, s) \times (\uparrow, \downarrow). \quad (11)$$

For the $q\bar{q}$ mesons this works well; we have

$$\mathbf{6} \otimes \mathbf{6}^* = \mathbf{1} + \mathbf{35}, \quad (12)$$

$$\mathbf{35} \rightarrow (\mathbf{8}, \mathbf{0}) + (\mathbf{1} + \mathbf{8}, \mathbf{1}). \quad (13)$$

Here the $\mathbf{8}$ and the $\mathbf{1}$ before the commas are the $SU(3)_f$ multiplicities and the $\mathbf{0}$ and the $\mathbf{1}$ after the commas are the spins of the particles. The octet of pseudoscalar mesons,

$$(K^+, K^0, \pi^+, \pi^0, \pi^-, \eta^0, \bar{K}^0, \bar{K}^-), \quad (14)$$

was known, as were the singlet plus octet (or nonet) of vector mesons,

$$(K^{*+}, K^{*0}, \phi^0, \rho^+ \rho^0, \rho^-, \omega^0, \bar{K}^{*0}, \bar{K}^{*-}). \quad (15)$$

Both the octet and the nonet fit well in the $SU(6)$ scheme.

The analogous calculation for the qqq baryons requires decomposing the product of three **6**'s into irreducibles of $SU(6)$,

$$\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6} = \mathbf{56} + \mathbf{70} + \mathbf{70} + \mathbf{20}. \quad (16)$$

This **56** is the representation that fits the data on the lowlying baryons,

$$\mathbf{56} \rightarrow (\mathbf{8}, 1/2) + (\mathbf{10}, 3/2), \quad (17)$$

since there is an octet of spin-1/2 baryons ,

$$(p^+, n^0, \Lambda^0, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^0, \Xi^-), \quad (18)$$

and a decuplet of spin-3/2 baryons,

$$(\Delta^{++}, \Delta^+, \Delta^0, \Delta^-, Y_1^{*+}, Y_1^{*0}, Y_1^{*-}, \Xi^{*0}, \Xi^{*-}, \Omega^-). \quad (19)$$

Gürsey and Radicati found a mass formula for these baryons that generalizes the Gell-Mann–Okubo mass formula for each $SU(3)$ multiplet and also gives a new relation between masses in the octet and the decuplet.

The **56** seemed like a compelling choice for the baryons in the quark model. However, this leads to a paradox: The permutation properties of the **56**, **70** and **20** are respectively symmetric, mixed and antisymmetric. Since the quarks should have spin 1/2, the spin-statistics theorem [10] requires that they should be fermions and occur in the antisymmetric **20** representation. The experimental data which places the baryons in the symmetric **56** representation conflicts with the spin-statistics theorem.

When I came to Princeton in the fall of 1964 there was a lot of excitement about the Gürsey-Radicati $SU(6)$ theory. Benjamin W. Lee gave me a preprint of a paper [11] on the ratio of the magnetic moments of the proton and neutron that he had written with Mirza A. Baqi Bég and Abraham Pais. They had calculated

this magnetic moment ratio using the group theory of $SU(6)$. I translated their result into the concrete quark model, assuming the quarks obey bose statistics in the visible degrees of freedom. Both the result, that the ratio is $-3/2$, and the simplicity of the calculation were striking.

Here is my version of that calculation: Represent the proton and neutron with spin up as

$$|p_{\uparrow}^+\rangle = \frac{1}{\sqrt{3}}u_{\uparrow}^{\dagger}(u_{\uparrow}^{\dagger}d_{\downarrow}^{\dagger} - u_{\downarrow}^{\dagger}d_{\uparrow}^{\dagger})|0\rangle, \quad (20)$$

$$|n_{\uparrow}^0\rangle = \frac{1}{\sqrt{3}}d_{\uparrow}^{\dagger}(u_{\uparrow}^{\dagger}d_{\downarrow}^{\dagger} - u_{\downarrow}^{\dagger}d_{\uparrow}^{\dagger})|0\rangle. \quad (21)$$

The $(u_{\uparrow}^{\dagger}d_{\downarrow}^{\dagger} - u_{\downarrow}^{\dagger}d_{\uparrow}^{\dagger})$ combination in parentheses serves as a “core” that carries zero spin and isospin, so that the third quark to the left of the parentheses carries the spin and isospin of the proton or neutron. The magnetic moment is then the matrix element $\mu_B = \langle B_{\uparrow}|\mu_3|B_{\uparrow}\rangle$, where $\mu_3 = 2\mu_0\Sigma_q Q_q S_q$, $Q_q = (2/3, -1/3, -1/3)$, the 2 is the g-factor of the quark, μ_0 is the Bohr magneton of the quark and Q_q are the quark charges in units of the proton charge. With this setup the magnetic moments can be calculated on one line,

$$\mu_p = 2\mu_0\frac{1}{3}\{2[\frac{2}{3} \cdot 1 + (-\frac{1}{3}) \cdot (-\frac{1}{2})] + [(-\frac{1}{3}) \cdot (\frac{1}{2})]\} = \mu_0. \quad (22)$$

The analogous calculation for the neutron gives

$$\mu_n = -\frac{2}{3}\mu_0. \quad (23)$$

The ratio is $\mu_p/\mu_n = -3/2$, which agrees with experiment to 3%. This leads to an estimate for the effective mass of the quark in the nucleon, $m_N/2.79 \approx 340 MeV/c^2$, which is consistent with present estimates of the constituent masses of the up and down quarks.

Previous calculations of the magnetic moments using pion clouds had failed. Nobody had realized that the ratio was so simple. In retrospect the calculation worked better than we would now expect, since it did not take account of quark-antiquark pairs and gluons. Nonetheless, for me the success of this simple calculation was a very convincing additional argument that quarks have concrete reality.

The paradox about the placement of the baryons in the **56** representation of $SU(6)$ was based on the spin-statistics theorem which states: Particles that have integer spin must obey Bose statistics, and particles that have odd-half-integer spin must obey Fermi statistics. I knew there is a generalization of the spin-statistics theorem that was not part of general knowledge in 1964: Particles that have integer spin must obey parabose statistics, and particles that have odd-half-integer spin must obey parafermi statistics [12]. Each family is labeled by an integer p ; $p = 1$ is the ordinary Bose or Fermi statistics.

I immediately realized that parafermi statistics of order 3 would allow up to 3 quarks in the same space-spin-flavor state without violating the Pauli principle, which would resolve the statistics paradox. To test this idea I suggested a model in which quarks carry order-3 parafermi statistics in [3]. **This was the introduction of the hidden charge degree of freedom now called color.**

With this resolution of the statistics paradox I was exhilarated. I felt that the new charge degree of freedom implicit in the parafermi model would have lasting value. I became convinced that the quark model and color were important for the theory of elementary particles. Not everybody shared my enthusiasm.

It is difficult now to grasp the level of rejection of these ideas in 1964 and even for the next several years. Quarks were received with skepticism in 1964. Color as a hidden charge carried by quarks was received with disbelief.

The reactions of two distinguished physicists illustrate this skepticism and disbelief. I gave a copy of my paper to J. Robert Oppenheimer and asked his opinion of my work when I saw him at a conference about a week later. He said, “Greenberg, it’s beautiful!,” which sent me into an excited state. His next comment, however, “But I don’t believe a word of it.” brought me down to earth. In retrospect I have two comments about these remarks of Oppenheimer. I was not discouraged, because I was convinced that my solution to the statistics paradox would have lasting value. Nonetheless, I was too intimidated by Oppenheimer to ask why he did not believe my paper.

Steven Weinberg, who contributed as much as anybody to the standard model of elementary particles, wrote in a talk on *The Making of the Standard Model* [14] “At that time [referring to 1967] I did not have any faith in the existence of quarks.”

The skepticism about quarks and color can be understood: Quarks were new. Nobody had ever observed a particle with fractional electric charge. Gell-Mann himself was ambiguous about their reality. In his paper he wrote “...It is fun to speculate...if they were physical particles of finite mass (instead of purely mathematical entities as they would be in the limit of infinite mass...A search... would help to reassure us of the non-existence of real quarks [1].” To add a hidden charge degree of freedom to the unobserved fractionally charged quarks seemed to stretch credibility to the breaking point at that time. In addition, parastatistics, with which the new degree of freedom was introduced, was unfamiliar.

Resolving the statistics paradox was not a sufficient test of color. I needed new predictions. I turned to baryon spectroscopy to construct a model of the baryons in which the hidden parafermi (color) degree of freedom takes care of the required antisymmetry of the Pauli principle. Then I could treat the quarks as bosons in the visible space, spin and flavor degrees of freedom, with the parastatistics taking care of the necessary antisymmetry. I made a table of the excited baryons in the model using s and p state quarks in the **56**, $L = 0^+$ and **70**, $L = 1^-$ supermultiplets.

I followed up this work with Marvin Resnikoff in 1967 [13]. This work has been continued by Richard H. Dalitz and collaborators, by Nathan Isgur and Gabriel Karl and by Dan-Olof Riska and collaborators, among others. The original fits to the baryons made in 1967 are surprisingly close to the current fits of 2008.

The only evidence for color from 1964 to 1969 was the baryon spectroscopy that I proposed in 1964. It was only in 1968 that the first rapporteur at an international conference accepted the parastatistics model for baryons as the correct model. By then the data on baryon spectroscopy clearly favored the new degree of freedom. In 1969, Steven Adler, John Bell and Roman Jackiw explained the $\pi \rightarrow \gamma\gamma$ decay rate using the axial anomaly with colored quarks. This gave the first additional evidence for quarks.

3 Introduction of the gauge theory of color

Explicit color $SU(3)$ was introduced in 1965 by Yoichiro Nambu [15] and by Moo-Young Han and Nambu [16]. The papers of Nambu and of Han and Nambu used

3 dissimilar triplets in order to have integer charges for the quarks. This is not correct, both experimentally and theoretically for reasons given above. However this paper includes the statement “*Introduce now eight gauge vector fields which behave as $(1,8)$, namely as an octet in $SU(3)$ ”* [16]. **This was the introduction of the gauge theory of color.** The **1** in the **$(1,8)$** refers to what we now call flavor and is the statement that the gauge vector fields, which we now call gluons, are singlets in flavor. The **8** was what Han and Nambu called $SU(3)$ (which we now call $SU(3)_{color}$) and states that the interaction between the quarks is mediated by an octet of gluons.

Other solutions to the statistics paradox, all of which failed, were (i) an antisymmetric ground state, favored by Dalitz, (ii) the idea that quarks are not real, so that their statistics is irrelevant, and (iii) other atomic models. Adding $q\bar{q}$ pairs leads to unseen “exploding $SU(3)$ states.”

The original version of the quark model did not consider “saturation,” why only the combinations qqq and $q\bar{q}$ occur in nature. In 1966 Daniel Zwanziger and I surveyed the existing models and constructed new models to see which models account for saturation [17]. The only models that worked were the parafermi model, order 3, and the equivalent 3 triplet or color $SU(3)$ models. The states that are bosons or fermions in the parafermi model, order 3, are in 1-to-1 correspondence with the states that are color singlets in the $SU(3)$ model. Thus the parastatistics and explicit color models are equivalent as a classification symmetry.

Some properties beyond classification agree in both models. The $\pi \rightarrow \gamma\gamma$ decay rate and the ratio $\sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ are the same in both cases, because it does not matter whether the quark lines in intermediate states represent Green component quarks or color quarks.

Properties that require gauge theory include (i) confinement, discussed by Weinberg, by Gross and Wilczek and by Harald Fritzsch, Gell-Mann, and Heinrich Leutwyler in 1973, which explains why isolated quarks are not observed, (ii) asymptotic freedom, found by David Politzer and by Gross and Wilczek in 1973, which reconciles the quasi-free quarks of the parton model with the confined quarks of low-energy hadrons, (iii) running of coupling constants and precision tests of QCD, (iv) jets in high-energy collisions, among other things.

4 Summary

The discovery of color resolved a paradox: quarks as spin-1/2 particles should obey fermi statistics according to the spin-statistics theorem and should occur in *anti-symmetric* states; however they occur in the *symmetric* **56** of the Gürsey-Radicati $SU(6)$ theory. I resolved this paradox in 1964 by introducing a new 3-valued hidden charge degree of freedom, color, via the parafermi model of quarks in which color appears as a classification symmetry and a global quantum number. I used this model to predict correctly the spectroscopy of excited states of baryons. The other facet of the strong interaction, gauged $SU(3)_{color}$, was introduced as a local gauge theory by Nambu and by Han and Nambu in 1965.

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